Lecture 2 - Risk and Financial Crises [January 14, 2011]

Chapter 1. Financial Crisis of 2007-2008 and Its Connection to Probability Theory [00:00:00]

Professor Robert Shiller: So, what I want to do this time is talk about probability. I don't think many of you have taken a course in probability theory. I don't take that as a prerequisite for this course, but I think that actually Probability theory is fundamental to the way we think about finance. So, I wanted to talk about that a little bit today. And I'm going to put it in a concrete context, namely, the crisis that the world has been through since 2007, and which we're still in at this point. It's a financial crisis that's bigger than any since the Great Depression of the 1930's. There's many different ways of thinking about a crisis like this. And I wanted to focus on one way that people think about it in terms of probability models. So, that's not the only way, it's not necessarily my favorite way of thinking about it. That's, I think, a good way of introducing our discussion of probability as it relates to finance.

So, let's just think about the crisis. Most people, when they talk about financial crises, they talk in terms of narrative, of a historical narrative. So, I'll give you a quick and easy historical narrative about the crisis. The crisis began with bubbles in the stock market, and the housing market, and also in the commodities market. Bubbles are--I will talk about these later, but bubbles are events, in which people get very excited about something, and they drive the prices up really high, and it's got to break eventually. And there was a pre-break around 2000 when the stock market collapsed around the world. All over the world, the stock markets collapsed in 2000. But then they came back again after 2003 and they were on another boom, like a roller coaster ride. And then they collapsed again. That's the narrative story.

And then, both the housing market [and the] stock market collapsed. And then, what happened is, we see a bunch of institutional collapses. So, we see, in 2007, failures in companies that had invested in home mortgages. And we see a run on a bank in the United Kingdom, Northern Rock. It was arrested, but it looked like 1930's all over again with the bank failure. We saw bank failures in the United States. And then, we saw international cooperation to prevent this from spreading like a disease. And then, we had governments all over the world bailing out their banks and other companies. So, a disaster was averted, and then we had a nice rebound. That's the narrative story, OK. And it makes it sound--and I'm going to come back to it, because I like the narrative story of the collapse.

But I want to today focus on something that's more in keeping with probability, with the way financial theorists think about it. And what financial theorists will think about is that actually it's not just those few big events. The crisis we got into was the accumulation of a lot of little events. And sometimes they accumulate according to the laws of probability into big events. And you are just telling stories around these accumulation of shocks that affected the economy. And the stories are not, by some accounts, not that helpful. We want to understand the underlying probabilities.

I'm going to talk today about probability, and variance, and covariance, and regression, and idiosyncratic risk, and systematic risk. Things like that which are core concepts in finance. But I'm also going to, in the context of the crisis, emphasize in this lecture, breakdowns of some of the most popular assumptions that underlie financial theory. And I'm thinking particularly of two breakdowns. And we'll emphasize these as other interpretations of the crisis. One is the failure of independence. I'll come back and redefine that. And another one is a tendency for outliers or fat-tailed distributions. So, I'll have to explain what all that means.

Chapter 2. Introduction to Probability Theory [00:05:51]

But basically, let me just try to elaborate on--probability theory is a conceptual framework that mathematicians invented. And it's become a very important way of thinking, but it doesn't go back that far in time. The word probability in its present meaning wasn't even coined until the 1600's. So, you if you talk to someone before the year 1600, and say, this has a probability of 0.5, they would have no idea what you're talking about. So, it's a major advance in human understanding to think in terms of probabilities. Now we do. And now it's routine, but it wasn't routine at all. And part of what I'm thinking about is, what probability theorists do, or in particular finance theorists like to do, is they think that the world is very complex, and that the outcomes that we see are the results of millions of little things. And the stories we tell are just stories.

So, how do we deal with the complexity of the world? Well, we do it by dealing with all of these little incremental shocks that affect our lives in a mathematical way. And we think of them as millions of shocks. How do they accumulate? We have mathematical laws of how they accumulate. And once we understand those laws, we can build mathematical models of the outcomes. And then we can ask whether we should be surprised by the financial events that we've seen. It's a little bit like science, real hard science.

So, for example, weather forecasters. They build models that--you know, you see these weather forecasts. They have computer models that are built on the theory of fluid dynamics. And there is a theory of all those little atoms moving around in the air. And there's too many atoms to count, but there's some laws about their cumulative movement that we understand. And it actually allows us to forecast the weather. And so, people who are steeped in this tradition in finance think that what we're doing when we're doing financial forecast is very much like what we do when we do weather forecasts. We have a statistical model, we see all of the shocks coming in, and of course there will be hurricanes. And we can only forecast them--you know there's a limit to how far out we can forecast them. So, all hurricanes are a surprise two weeks before they happen. Weather forecasters can't do that. Same thing with financial crises. This would be the model. We understand the probability laws, there's only a certain time horizon before which we can forecast the financial crisis.

Chapter 3. Financial Return and Basic Statistical Concepts [00:09:58]

I want to start out with just the concept of return. Which is, in finance, the basic, the most basic concept.

When you invest in something, you have to do it for a time interval. And I'm writing the return as one time period. T is time. And so, it could be [a] year, or it could be months, or it could be [a] day. Let's say it's monthly return, we're going to number these months, so that the first month is number one, second month is number two.

And so, return at time t, if t is equal to 3, that would be the return at month three. And we'll do price at the beginning of the month. And so, what is your return to investing in something? It's the increase in the price. That's $p_{t+1} - p_t$. which is called the capital gain, plus the dividend, which is a check you receive, if you do, from the company that you're investing in. That's the return.

We have something else called gross return. Which is just 1 plus the return. Returns can be positive or negative. They can never be less than minus 100%. In a limited liability economy that we live in, the law says that you cannot lose more than the money you put in, and that's going to be our assumption. So, return is between minus 100% and plus infinity. And gross return is always positive. It's between zero and infinity.

I want to now talk about some basic statistical concepts that we can apply to returns and to other random variables as well. This is expected value. This is the mathematical expectation of a random variable x, which could be the return, or the gross return, or something else.

We're going to substitute in what they are. So, the expectation of x, or the mean of x, μ_x is another term for it, is the weighted sum of all possible values of x weighted by their probabilities. And the probabilities have to sum to 1. They're positive numbers, or zero or positive numbers, reflecting the likelihood of that value of the random variable occurring.

So, I have here--there's an infinite number of possible values for x, and we have a probability for each one, and the expectation of x is that weighted sum of those, weighted by probabilities, of those possible values. This is for a discrete random variable that takes on only a countable number of values. if x is continuous, then the expectation of x is an integral of the probability density of x, times x dx:

…

These two formulas here are measures of the central tendency of x. This formula is something we use to estimate the expected value of x. This is called the mean or average, which you've learned long ago. If you have n observations on a random variable x, you can take the sum of the x observations, summation [over] i equals 1 to n, and then divide that by n. That's called the average. So, what I want to say is that this is the average, or the mean, or sample mean when you have a sample of n observations, which is an estimate of the expected value of x.

So, for example, if we're evaluating an investor who has invested money, you could get n observations, say annual returns, and you can take an average of them. And that's the first and most obvious metric representing the success of the investments if x is the return. People are always wanting to know, they're looking at someone who invests money, is this person a success or not? Well this is the first and most obvious measure. Let's see what that person did on average. You were investing for, let's say n equals 10, ten years, let's take the returns you made each year, add them up and divide by 10. And that gives us an average.

I put a formula down as an alternative called the geometric mean. The former was the arithmetic mean. This is the geometric mean and it's a different concept. The geometric mean, instead of adding your n observations, you multiply them together. You form a product of them. And then, instead of dividing by n, you take the nth root of the product. And so, that's a formula that's used to estimate the average return of a portfolio of investments, where we use gross return for x, not just the simple return. This geometric mean makes sense only when all the x's are non-negative. If you put in a negative value, you might get a negative product, and then, if you took the nth root of that, it could be an imaginary number, so let's forget that. We're not going to apply this formula if there are any negative numbers. But it's often used, and I recommend its use, in evaluating investments. Because if you use gross return, it gives a better measure of the outcome of the investments.

So, think of it this way. Suppose you invested money with some investment manager, and the guy said, I've done a wonderful job investing your money. I made 50% one year, I made 30% another year, oh, and by the way, I had one bad year with minus 100%. What do you think of this investor? Well, you think about it, if he made 50% one year, and then 30% another year, and then he lost everything. That dominates everything, right? If you have a minus 100% simple return, your gross return is 0, OK? If I plug in, if I put in a 0 here to any of the x's, right, this product will be 0. Anything times 0 is 0. And I take the nth root of zero, and what's that? It's 0. So, if there's ever a year in which the return is minus 100%, then the geometric mean is 0. That's a good discipline. The arithmetic mean obviously doesn't make sense as a way to evaluate investment success.

These are all measures of central tendency. That is, what is the central result? Sometimes the investor had a good year, sometimes the investor had a bad year, but what was the typical or central value? So, these are a couple of measures of them. But we care more than just about central tendency when evaluating risk. We have to do other things as well. This is very fundamental to finance. We have to talk about risk. What could be more fundamental than risk for finance? So, what we have here now is a measure of variability. And the upper equation here is something called variance. And it's equal to the weighted average of the x random variables' [correction: realizations of the x random variables'] squared deviation from the mean, weighted by probabilities. All it is, is the expectation of the square of the deviation from the mean. The mean is the center value, and the deviations from the mean are--whether they're positive or negative, if you square them they become positive numbers. And so, that's called variance. This is a very simple concept. It's just the average squared deviation from the mean. The estimate of the variance, or the sample variance, is given by this equation.

The next thing is covariance. They're very basic concepts. Covariance is a measure of how two different random variables move together. So, I have two different random variables, x and y. So, x is the return on, let's say, the IBM Corporation, and y is the return on General Motors Corporation. And I want to know, when IBM goes up, does General Motors go up or not? So, a measure of the co-movement of the two would be to take the deviation of x from its mean times the deviation of y from it's mean, and take the average product of those. And that's called covariance.

It's a positive number if, when x is high relative to it's mean, y is high relative to its mean also. And it's a negative number if they tend to go in opposite directions. If GM tends to do well when IBM does poorly, then we have a negative covariance. Because, if one is above its mean, and the other is below its mean, the product is going to be a negative number. If we get a lot of negative products like that, it means that they tend to move opposite each other. And if they are unrelated to each other, then the covariance tends to be 0.

If x and y are independent, they're generated [independently]-- suppose IBM's business has just nothing at all to do with GM's businesses, they're so different. Then I'd say the covariance is probably 0. And then we can use that as a principle, which will underlie our later analysis.

Correlation is a scaled covariance. And it's a measure of how much two variables move together. But it's scaled, so that it varies only over the range of minus 1 to plus 1. So, the correlation between two random variables is their covariance divided by the product of their standard deviations. And you can show that that always ranges between minus 1 and plus 1. So, if two variables have a +1 correlation, that means they move exactly together. When one moves up 5%, the other one moves up 5% exactly. If they have a correlation of -1, it means the move exactly opposite each other. These things don't happen very often in finance, but in theory that's what happens. If they have a zero correlation, that means there's no tendency for them to move together at all. If two variables are independent, then their correlation should be zero.

OK, the variance of the sum of two random variables is the variance of the first random variable, plus the variance of the second random variable, plus twice the covariance of the random variables. So, if the two random variables are independent of each other, then their covariance is zero, and then the variance of the sum is the sum of the variances. But that is not necessary. That's true if the random variables are independent, but we're going to see that breakdown of independence is the story of this lecture right now. We want to think about independence as mattering a lot. And it's a model, or a core idea, but when do we know that things are independent?

Chapter 4. Independence and Failure of Independence as a Cause for Financial Crises [00:26:29]

That was a plot of the stock market from 2000 to 2010 in the U.S. These are the crises I was telling you about. This is the decline in the stock market from 2000 to 2002 or 2003 and this is the more recent decline from 2007 to 2009. Those are the cumulative effects of a lot of little shocks that didn't happen all at once. It happened over years. And we want to think about the probability of those shocks occurring. And that's where I am going.

But what I want to talk about is the core concept of independence leading to some basic principles of risk management. The crisis that we've seen here in the stock market is the accumulation of--you see all these ups and downs in the stock market, and then all these ups and downs on the way up. There were relatively more downs in the period from 2000 and 2002 and there were relatively more ups from the period 2003 to 2006. But how do we understand the cumulative effect of it, which is what matters? So, we have to have some kind of Probability Model. The question immediately is, are these shocks that affected the stock market, are they independent, or are they somehow related to each other? And that is a core question that made it so difficult for us to understand how to deal with the potential of such a crisis, and why so many people got in trouble dealing with this crisis.

We had a big financial crisis in the United States in 1987, when there was a stock market crash that was bigger than any before in one day. But after the 1987 crash, companies started to compute a measure of the risk to their company, which is called Value at Risk. And what companies would do after 1987 to try to measure the risk of their activities is to compute a number something like this. They would say, there's a 5% probability that we will lose $10 million in a year. That's the kind of bottom line that Value at Risk calculations would make.

You need a Probability Model to make these calculations. And so, you need probability theory in order to do that. Many companies had calculated Value at Risk numbers like this, and told their investors, we can't do too badly because there's no way that we could lose--the probability is only 5% that we could lose $10 million. And they'd have other numbers like this. But they were implicitly making assumptions about independence, or at least relative independence. And that's the concept I'm trying to emphasize here. It's a core concept in finance. And it's not one that is easy to be precise about. The problem that brought us this crisis is that the Value at Risk calculations were too optimistic. Companies all over the world were estimating very small numbers here, relative to what actually happened. And that's a problem.

I wanted to emphasize core concepts here. One of these concepts is something we'll call the law of large numbers. And the law of large numbers says that, there's many different ways of formulating it, but putting it in its simplest form, that if I have a lot of independent shocks, and average them out, on average there's not going to be much uncertainty. If I flip a coin once, let's say I'm making a bet, plus or minus. If it comes up heads, I win a dollar; if it comes up tails, I lose a dollar. Well, I have a risk. I mean, I have a standard deviation of $1 in my outcome for that. But if I do it 100 times and average the result, there's not going to be much risk at all. And that's the law of large numbers. It says that the variance of the average of n random variables that are all independent and identically distributed goes to 0 as the number of elements in the average goes to infinity. And so, that's a fundamental concept that underlies both finance and insurance. The idea that it has uncertainty in a small number of observations, but the uncertainty vanishes in a large number of observations, goes back to the ancient world. Aristotle made this observation, but he didn't have probability theory and he couldn't carry it further.

The fundamental concept of insurance relies on this intuitive idea. And the idea was intuitive enough that insurance was known and practiced in ancient times. But the insurance concept depends on independence. Independence is something that apparently breaks down at times like big down crises that we've seen in the stock market, in the two episodes in the beginning of the 20th century.

The law of large numbers has to do with the idea that if I have a large number of random variables, what is the variance of--the variance of $x_1 + x_2 + x_3 + \ldots + x_n$? If they're all independent, then all of the covariances are 0. So, it equals the variance of x_1, plus the variance of x_2, … , plus the variance of x_n. If they all have the same variance, then the variance of the sum of n of them is n times their variance. That means the standard deviation, which is the square root of the variance, is equal to the square root of n times the standard deviation of one of them. The mean is divided by n. So, that means that the standard deviation of the mean is equal to the standard deviation of one of the x's divided by the square root of n. So, as n goes large, you can see that the standard deviation of the mean goes to 0. And that's the law of large numbers.

But the problem is, so you know, you can look at a financial firm, and they have returns for a number of years, and those returns can be cumulated to give some sense of their total outcome. But does the total outcome really behave properly? Does it become certain over a longer interval of time? Well, apparently not, because of the possibility that the observations are not independent.

We want to move from analysis of variance to something that's more--I told you that VaR came in 1987 or thereabouts, after the stock market crash of '87. There's a new idea coming up now, after this recent crisis, and it's called CoVaR. And this is a concept emphasized by Professor Brunnermeier at Princeton and some of his colleagues, that we have to change analysis of variance to recognize, I'm sorry, we have to change Value at Risk to recognize that portfolios can sometimes co-vary more than we thought. That there might be episodes when everything goes wrong at the same time. So, suddenly the covariance goes up. So, CoVaR is an alternative to Value at Risk that does different kinds of calculations. In the present environment, I think, we recognize the need for that.

Chapter 5. Regression Analysis, Systematic vs. Idiosyncratic Risk [00:38:58]

Let me go to another plot which shows both the same aggregate stock market, that's this blue line down here, and one stock. The one stock I have shown is Apple, the computer company. And this is from the year 2000to 2010. The stock market lost something like almost half of its value. It dropped 40% between 2000 and 2002. Wow. Then it went all the way back up, and then it dropped almost 50%. These are scary numbers, right? But when I put Apple on the same plot, the computer had to, because Apple did such amazing things, it had to compress. And that's the same curve that you were just looking at. It's just compressed, so that I can plot it together. I put both of them at 100 in the year 2000. So, what I'm saying here is that somehow Apple did rather differently than the--this is the S&P 500. It's a measure of the whole stock market. Apple computer is the one of the breakout cases of dramatic success in investing. It went up 25 times.

This incidentally is the adjusted price for Apple, because in 2005 Apple did a 2-for-1 split. You know what that means? By tradition in the United States, stocks should be worth about $30 per share. And there's no reason why they should be $30 per share. But a lot of companies, when the price hits $60 or something like that, they say, well let's just split all the shares in two. So, that they're back to $30. Apple went up more than double, but they only did one split in this period. So, we've corrected for that. Otherwise, you'd see a big apparent drop in their stock price on the day of the split. Are you with me on this split thing? It really doesn't matter, it's just a units thing. But you can see that an investment in Apple went up 25 times, whereas an investment in the S&P 500 went up only--well, it didn't go up, actually, it's down.

So now, this is a plot showing the monthly returns on Apple. It's only the capital gain returns; I didn't include dividends. But it is essentially the return on these two, on the S&P 500 and on Apple. Now, this is the same data you were just looking at, but it looks really different now, doesn't it? It looks really different. They're unrecognizable as the same thing. You can't tell from this plot that Apple went up 25-fold. That matters a lot to an investor. Maybe you can, if you've got very good eyes. There's more up ones than there are down ones, more up months than down months. There's a huge number of--enormous variability in the months.

But I like to look at a picture like this, because it conveys to me the incredible complexity of the story. What was driving Apple up and down so many times? Really a pretty simple picture. Buy Apple and your money will go up 25-fold. Incidentally, if you were a precocious teenager, and you told your parents ten years ago, mom, let's take out a $400,000 mortgage on the house and put it all in Apple stock. Your parents would thank you today if you told them to do that. Your parents could do that. They have probably paid off their mortgages, right; they could go get a second mortgage. Easily come up with $400,000. Most of your houses would be worth that. So, what would it be worth today? $10 million. Your father, your mother would be saying, you know, I've been working all ten years, and your little advice just got me $10 million. It's more than I made, much more than I made in all those years.

These kinds of stories attract attention. But you know, it wasn't an even ride. That story seems too good to be true, doesn't it? I mean 25-fold? The reason why it's not so obvious is that the ride, as you're observing this happen, every month it goes opposite. It just goes [in] big swings. You make 30% in one month; you lose 30% in another month. It's a scary ride. And you can't see it happening unless you look at your portfolio and see what--you can't tell. It's just so much randomness from one month to the other.

Incidentally, I was a dinner speaker last night for a Yale alumni dinner in New York City. And I rode in with Peter Salovey who's Provost at Yale. And on the ride back he reminded me of a story that I think I've heard, but it took me a while to remember this. But I'll tell you that it's an important Yale story. And that is that in 1979, the Yale class of 1954 had a 25th reunion. Somebody said, you know, we're here at this reunion, there's a lot of us here, let's all, as an experiment, chip in some money and ask an investor to take a risky portfolio investment for Yale and let's give it to Yale on our 50th anniversary, all right? Sounds like fun.

They got a portfolio manager, his name was Joe McNay, and they said--they put together--it was $375,000. It's like one house, you know, for all the whole class of 1954, no big deal. So, they gave Joe McNay a $375,000 start. And they said, just have fun with this. You know, we're not conservative. If you lose the whole thing, go ahead. But just go for maximum return on this.

Joe McNey decided to invest in Home Depot, Walmart, and Internet stocks. And on their 50th reunion, that was 2004, they presented Yale University with $90 million dollars. That's an amazing story. But I'm sure it was the same sort of thing, same kind of roller coaster ride the whole time. And now, we're trying to decide, is Joe McNey a genius? What do you think, is he a genius? I think, maybe he is. But the other side of it is, I just told you what to do in just a few words. It's Walmart Home Depot, and Internet stocks. And the other thing is, he started liquidating in 2000, right the peak of the market. So, it must be partly luck.

How did he know that Walmart was a good investment in 1979? I don't know. It's sort of--he took the risks. Maybe he is just lucky. No one could have known that Walmart was going to be such a success.

And I think that history is like that. The people you read about in history, these great men and women of history, are often just phenomenal risk takers like Joe McNey. And for every one of them that you read about, there's 1,000 of them that got squashed. I was reading the history of Julius Caesar, as written by Plutarch. This guy is a real risk taker. He just went for it every time. And he ended up emperor of Rome. But you know what happened to him, he got assassinated. It turned out not entirely a happy story. So, maybe it's all those poor, all those ordinary people, who live in the little house, the $400,000 house, they don't risk it. Maybe they're the smart ones. You just don't ever hear of them.

Well, these are issues for finance. But you wonder, what are all of these things, all of these big movements? This is the worst one here, where it lost about a third of its value in one month. And I researched it. What was it? Does anyone know what caused it in 2008? Well, I'll tell you what caused Apple to lose a third of its value in one month. Steve Jobs, who is the founder of Apple and genius behind the company, gave a--was at an annual meeting or press conference, and people said, he doesn't look well. And so, they recalled that he had pancreatic cancer in 2004, but the doctors then said it's curable, no problem, so the stock didn't do anything. But reporters called Apple and said, is he ok? And their company spokesman wouldn't say anything. So, it started a rumor mill that Steve Jobs was dying of cancer. It quickly rebounded because he wasn't. That's how crazy these things are, these market movements.

Now the next plot, and this is important for our concepts here. I can plot the same data in different ways. This shows a different sort of complexity. Let me just review what we've seen here. We started out with Apple stock. This is the stock price normalized to 100 in 2000. It goes up to 2500. Then, the next thing I did is I did capital gains as a percent. The percentage increase in price for each month. It looks totally different, and it shows such complexity that I can't tell a simple narrative. I've just told you about one blip here, but they were so many of these blips on the way, and they all have some story about the success of some Apple product, or people aren't buying some product. Every month looks different. What I want to do--and I have here the blue line is the return of the S&P 500. Now what I want to do is plot a different sort of plot. It's a scatter plot. I'm going to plot the return on Apple against the return on the S&P 500. This is scatter plot. On the vertical axis I have the return, it's actually the capital gain on Apple, and on the horizontal axis I have the capital gain on the whole stock market. Each point represents one of the points that we saw on the market. Actually I think it was, I was telling you the second lowest return. One of these points in 2008 was when Steve Jobs looked sick. So, each point is a month, and I have the whole decade of 2000, of the beginning of the 2000s, plotted.

So, the best success was in December, January of 2001, where the stock price went up 50% in one month. I tried to figure out what that was about. Why'd they go up 50% percent in one month? It turns out that the preceding two months it had gone down a lot. They were down here somewhere. There were these big drops, and people were getting really pessimistic because Apple products weren't going well. They had introduced some new products, Mobile Me, I think, we forget about these products that don't work, that didn't work very well. And then somehow people decided it really wasn't so bad, so we have plus 50, almost 50% return in one month. The reason why it looks kind of compressed on this way is, because the stock market doesn't move as much as Apple.

Basically Apple return is the sum of two components, which is the overall market return, and the idiosyncratic return. The return for a stock, for the i-th stock, is equal to the market return, which is represented here by the S&P 500, which is pretty much the whole stock market, plus idiosyncratic return. If they're independent of each other, the variance of the sum is the sum of the variance. The variance of the stock returns is the variance--the variance of the Apple return is the sum of their market return and their idiosyncratic return.

Let's add a regression line to the scatter plot. It's the same scatter that you saw--is it clear? Everyone clear what we're doing here? I've got S&P on this axis, and Apple on this axis. And now I've added a line, which is a least-square fit, which minimizes the sum of squared deviations from the line. It tries to get through the scatter of points as much as it can. And the line has a slope of 1.45. We call that the β, all right? It's a simple idea here. What it means is that it seems like Apple shows a magnified response to the stock market. It goes up and down approximately one and a half times as much as the stock market does on any day. The Apple return here is equal to the β times the return on the S&P that you see here.

Why does Apple respond more than one-for-one with the stock market? I guess it's because the aggregate economy matters, right? If you think that maybe because Apple is kind of a vulnerable company, that if the economy tanks, Apple will tank even more than the economy, than the aggregate economy, because they're such a volatile, dangerous strategy company. And if the market goes up, then it's even better news for Apple. But even so, the idiosyncratic risk just dominates. Look at these observations, way up and way down here. Apple has a lot of idiosyncratic risk. And I mentioned one example; it's Steve Jobs' health.

The Steve Jobs story is remarkable. He founded Apple and Apple prospered, and then he kind of had a falling out with the management, and got kind of kicked out of his own company. And then he says, all right, I'll start my own computer company, my second, I'll do it again. So, he founded Next Computer. But meanwhile, Apple started to really tank. This is in the nineties. And they finally realized they needed Steve Jobs, so they brought him back. So, the company's ups and downs, the idiosyncratic risk, has a lot to do with Steve Jobs, and what he does, the mistakes he made. Those are what causes these big movements.

This line, I thought it would have an even higher β. But I think it's this point which is bringing the β down. And this is, I think this is the point--the month after it turns out that Steve Jobs really wasn't sick. And it turned out to be the same month that's the Lehman Brothers collapse occurred. So you see, this point here is between September and October of 2008. And that's the point--it was September 15th that we had the most significant bankruptcy in U.S. history. Lehman Brothers, the investment bank, went bankrupt. It threw the whole world in chaos. So, the stock market and S&P 500 stock market return was minus 16% in one month, horrible drop. But for Apple, it really was only about minus 5%, because they're getting over the news of Steve Jobs. So, that's the way things work.

Chapter 6. Fat-Tailed Distributions and Their Role during Financial Crises [00:58:59]

I want to move on now to next topic, which is outliers, and talk about another assumption that is made in finance traditionally that turned out to be wrong in this episode. The assumption is that random shocks to the financial economy are normally distributed. You must have heard of the normal distribution. This is the bell-shaped, the famous bell-shaped curve, that was discovered by the mathematician Gauss over a hundred years ago. The bell-shaped curve is thought to be the log of this curve is a parabola. It's a particular mathematical function. The curve is thought by statisticians to recur in nature many different ways. It has a certain probability law.

I have plotted two normal distributions, and I have them for two different standard deviations. One of them, black line, is the standard deviation of 3, and the other one, the pink line, is the standard deviation of 1. But they both look the same; they're just scaled differently. And these distributions have the property that the area under the curve is equal to 1 and the area between any two points, say between minus 5 and minus 10, the area under this curve is the probability that the random variable falls between minus 5 and minus 10.

A lot of probability theory works on the assumption that variables are normally distributed. But random variables have a habit of not behaving that way, especially in finance it seems. And so, we had a mathematician here in the Yale math department, Benoit Mandelbrot, who was really the discoverer of this concept, and I think the most important figure in it. [Correction: Pierre Paul Levy invented the concept, as discussed in the next lecture.] He said that in nature the normal distribution is not the only distribution that occurs, and that especially in certain kinds of circumstances we have more fat-tailed distributions. So, this blue line is the normal distribution, and the pink line that I've shown is a fat-tailed distribution that Mandelbrot talked about, called the Cauchy distribution.

You see how it differs? The pink line looks pretty much the same. They're both bell-shaped curves, right? But the pink line has tremendously large probability of being far out. These are the tails of the distribution. So, if you observe a random variable that looks--you observe it for a while, maybe you get 100 observations, you probably can't tell it apart very well from a normal distribution. Whether it's Cauchy or normal, they look about the same. The way you find out that they're not the same, is that in extremely rare circumstances there'll be a sudden major jump in the variable that you might have thought couldn't happen.

I have here a plot of a histogram of stock price movements from 1928, every day, I've taken every day since 1928, and I've shown what the S&P Composite Index--it didn't have 500 stocks in 1928, so I can't call it S&P 500 for the whole period--but this is essentially the S&P 500. And I have every day. There's something like 40,000 days. And what this line here shows is that the stock return, the percentage change in stock price in one day, was between 0 and 1% over 9,000 times. And it was between 0 and minus 1 percent around 8,000 times. OK? So, that's typical [per] day. You know, it's less than 1% up or down. But occasionally, we'll have a 2% day. This is between 1 and 2% that occurred about 2,000 times. And about 2,000 times we had between minus 1 and minus 2%. And then, you can see that we've had--you can see these outliers here. These look like outliers, they're not extreme outliers. So, if you look at a small number of data, you get an impression that well, you know, the stock market goes up between plus or minus 2%, usually not so much, that's the way it is.

After here they don't seem to be anything, which means that, it looks like you never see anything more than up or down 5% or 6%. It just doesn't happen. Well, because it's so few days that it does those extremes. Can you see these little--that's between 5 and 6. There were maybe like 20 days, I can't read off the chart when it did this since 1928. You can go through ten years on Wall Street and never see a drop of that magnitude. So, eventually you get kind of assured. It can't happen. What about an 8% drop? Well, I look at this, I say, I've never seen that. You know, I've been watching this now, I've seen thousands of days, and I've never seen that. But I have here the two extremes. Stock market went up 12.53% on October 30, 1929. That's the biggest one-day increase. That's way off the charts, and if you compute the normal distribution, what's the probability of that? If it's a normal distribution and it fits the central portion, it would say it's virtually zero. It couldn't happen. Anyone have any idea what happened on October 30, 1929? It's obvious to me, but it's not obvious to you. I'm asking you to--I won't ask. What happened in October, anyone know what happened in October 1929?

Student: That must be right before the crash.

Professor Robert Shiller: You're close. You're right. But someone else?

Student: Wasn't it the rebound after the crash?

Professor Robert Shiller: Yes, absolutely, it was the rebound after the crash. The stock market crash of 1929 had two consecutive days. Boy is that probability, independence doesn't seem right. It went down about 12% on October 28, and then the next day it did it again. What's going on here? We were down like 24% in two days. People got up on the 30th and said, oh my God, is it going to do that again? But it did just the opposite. It was going totally wild. So, we don't know whether covariance broke down or not. I guess it didn't, because it rebounded, and that was the biggest one-day increase ever.

But if that weren't enough, however, let's go back to October 19, 1987. It went down 20.47% in one day. It went down even more on the Dow. Some people say it went down more than that, didn't it? But on the S&P that's how much it went down. So, I figured, well if this were normally distributed with the standard deviation suggested by this, what's the probability of a decline that's that negative? It's 10^{-71}.. So, you take 1 and you divide that by 1 followed by seventy-one zeross. That's an awfully small number. If you believe in normality, October 19, 1987 couldn't happen. But there it is. It happened.

And in fact, I was, I told you I've been teaching this course for 25 years. I was giving a lecture, not in this room, but nearby here, and I was talking about something else. And a student had a transistor radio. Remember transistor radios? And he was holding it up and listening to it. Then he raised his hand and said, do you know what's happening? He said the stock market is totally falling apart. It just came as a complete surprise to me.

So, after class, I didn't go back to my office. I went downtown to Merrill Lynch. And I walked up; it's a story I like to tell. It's not that good. I walked up and I talked to a stockbroker there, and I said, I was about to say something, but he didn't let me talk. He said, don't panic. He thought that I had shown up as someone who was losing everything, his life savings all in one day. And he said, don't worry, it's not going to--it's going to rebound. It didn't rebound. I showed up at lunchtime and it kept going down.

So, anyway, there was something wrong with independence. Let me just recap. The two themes are that independence leads to the law of large numbers, and it leads to some sort of stability. Either independence through time or independence across stocks. So, if you diversify through time or you diversify across stocks, you're supposed to be safe. But that's not what happened in this crisis and that's the big question. And then it's fat-tails, which is kind of related. But it's that distributions fool you. You get big incredible shocks that you thought couldn't happen, and they just come up with a certain low probability, but with a certain regularity in finance. All right, I'll stop there. I'll see you on next Wednesday.